PERFORMANCE RANKING OF PLATE-FINNED
HEAT EXCHANGER SURFACES

James Gilbert Soland

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by

JAMES GILBERT SOLAND

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ABSTRACT

The Kays-London (4) way of presenting heat exchanger performance is modified to use the data in the simplified analysis of La Haye et al. (5) which was extended to permit the ready comparison of the performance of the various plate-finned surfaces. Based on the four comparison criteria considered, the wavy finned 17.8 - 3/8 W surface of Kays-London is the best.

THESIS SUPERVISOR: Professor Warren M. Rohsenow
TITLE: Professor of Mechanical Engineering

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NOMENCLATURE

Symbol	Definition	Units
a	heat exchanger width	ft
A_b	heat transfer area of base surface ignoring any enhancement; equals length times heated perimeter	ft^2
A_c	minimum free flow area	ft^2
A_f	frontal area of heat exchanger core	ft^2
A_F	flow area ignoring any enhancing surfaces	ft^2
A_T	total heat transfer area	ft^2
b	plate spacing	ft
c_p	specific heat	Btu/lbm-^oF
D_n	nominal diameter; defined by (1b)	ft
e	roughness height for a granular surface	ft
f	friction factor based on total area (A_T); defined by (4a)	——
f_n	friction factor based on base area (A_b); defined by (4b)	——
f_s	friction factor for a smooth surface; defined by (29)	——
g_o	conversion factor (= 32.174 lbm-ft/lbf-sec^2)	——
G_c	mass flux based on minimum free flow area; defined by (2a)	lbm/hr-ft^2

G_n	mass flux based on free flow area (A_F); defined by (2b)	$lbm/hr\text{-}ft^2$
h	heat transfer coefficient based on total area (A_T); defined by (5a)	$Btu/hr\text{-}ft^2\text{-}{}^oF$
h_n	heat transfer coefficient based on base area (A_b); defined by (5b)	$Btu/hr\text{-}ft^2\text{-}{}^oF$
j	Colburn j-Factor based on total area (A_T); defined by (7a)	- -
j_n	Colburn j-factor based on base area (A_b); defined by (7b)	- -
j_s	Colburn j-factor for smooth surface; defined by (27)	- -
k	thermal conductivity	$Btu/hr\text{-}ft\text{-}{}^oF$
ℓ	fin length from root to center (=b/2)	ft
L	heat exchanger length	ft
m	component of fin efficiency (η_f); defined by (10)	- -
NTU	number of transfer units; defined by (21)	- -
p	pressure	lbf/ft^2
P	pumping power	hp
q	heat transfer rate	Btu/hr
q/A	heat flux	$Btu/hr\text{-}ft^2$
r_h	hydraulic radius; defined by (1a)	ft
T	temperature	${}^oF$
U	overall heat transfer coefficient	$Btu/hr\text{-}ft^2\text{-}{}^oF$
V	heat exchanger volume on one side	ft^3

DIMENSIONLESS GROUPS

Nu	Nusselt number; defined by (6a)	- -
Nu_n	Nusselt number; defined by (6b)	- -
Pr	Prandtl number	- -
Re	Reynolds number based on minimum free flow area (A_c); defined by (3a)	- -
Re_n	Reynolds number based on free flow area (A_F); defined by (3b)	- -

SUBSCRIPTS

a	case a parameter (Shape, V = const.)	- -
b	case b parameter (P, V = const.)	- -
c	case c parameter (NTU, P = const.)	- -
d	case d parameter (NTU, V = const.)	- -
e	enhanced surface	- -
m	heat exchanger metal	- -
s	smooth surface	- -

MISCELLANEOUS

β	ratio of total heat transfer area (A_T) to volume (V)	ft^{-1}
Δp_F	friction pressure drop	lbf/ft^2
ω	mass flow rate	lbm/hr
η_f	fin efficiency; defined by (9)	- -
η_o	total surface temperature effectiveness; defined by (8)	- -
μ	viscosity	lbm/hr-ft
ρ	density	lbm/ft^3
ε	heat exchanger effectiveness	- -
δ	fin thickness	ft
ψ	pin diameter	ft

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I. INTRODUCTION

The goal of any heat exchanger comparison method is to enable the designer to select from among the various enhanced surfaces that surface which is most beneficial. The comparison method should allow this selection to be made as easily and as accurately as possible.

Enhanced surfaces, particulary in the form of plate-fins, have been used in heat exchangers for many years. Attempts at comparing performance of various surfaces have also been made for many years. The more recent of these are by Bergles et al. (1), (2).

La Haye et al. (5) developed a method of comparing surfaces and used it to show how an effective uninterrupted flow length to diameter ratio could parametrically order the performance of surfaces. Here their comparison method is modified and used to compare the performance of all of the Kays-London (4) plate finned surfaces, unfinned surfaces and sand roughened surfaces (3). The method is universally applicable to any other heat exchanger surface as well.

II. COMPARISON METHOD

To simplify the comparison method, consider the performance of only one side of a plate-finned heat exchanger. This is equivalent to considering a heat exchanger with the controlling heat transfer resistance on one side - e.g., gas flow on the side of interest and condensing or boiling fluid on the other. We compare the performance of various finned and unfinned surfaces for the following quantities being the same:

1. ω , flow rate
2. $T_{h, in}$, hot fluid inlet temperature
3. $T_{c, in}$, cold fluid inlet temperature

Note also that the heat transfer resistance of the plate separating the two sides of the heat exchanger shall be considered to be negligible.

Kays and London (4), hereafter referred to as K-L, present data for many plate-finned surfaces in terms of heat transfer coefficients, h, and friction factors, f, referred to the exposed area, A_T, as a function of Reynolds number, Re, based on the minimum free flow area, A_c.

The proposed comparison method converts these h and f magnitudes to the base plate area, A_b; hence, the effect of the fins is included in the new h_n and f_n based on A_b. Further, the new Reynolds number, Re_n, will be based on the open flow, A_F, as though the fins were not present. This requires that the metal conductivity of the fins be

specified in incorporating the effect of the fins into the h_n.

The comparison of plate-finned performance is then converted to the same base that is currently used in comparing enhanced surfaces (such as those utilizing roughness, turbulence promoters, etc.) with plain, smooth surfaces.

Table I shows the proposed new definitions of the various quantities compared with the definitions used by K-L (4). Note from Figure 1, $D_n = 2b$ for either a finned or unfinned parallel plate passage.

To convert data of K-L to the new basis, the following ratios are obtained from the definitions, eqs. (1) through (10), and Figure 1.

$$\frac{A_b}{A_T} = \frac{2a\ L}{\beta\ V} = \frac{2}{\beta\ b} \tag{11}$$

where $\beta \equiv A_T/V$

$$\frac{A_F}{A_c} = \frac{L\ a\ b}{A_T\ r_h} = \frac{1}{\beta\ r_h} \tag{12}$$

$$\frac{G_n}{G_c} = \frac{A_c}{A_F} = \beta\ r_h \tag{13}$$

$$\frac{Re_n}{Re} = \frac{D_n\ G_n}{4\ r_h\ G_c} = \frac{\beta\ b}{2} \tag{14}$$

$$\frac{f_n}{f} = \frac{A_F\ A_T\ G_c^2}{A_c\ A_b\ G_n^2} = \frac{b}{2\ \beta^2\ r_h^3} \tag{15}$$

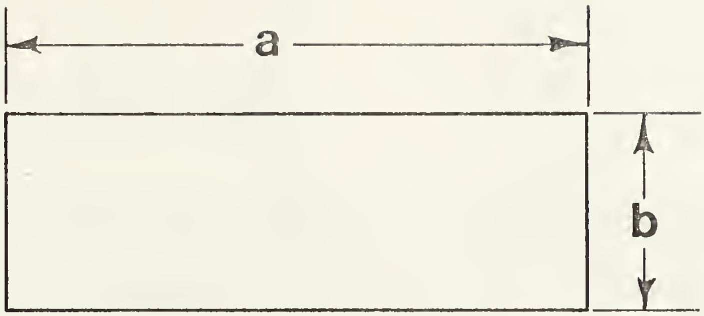
TABLE I. DEFINITIONS.

Quantity	Kays & London (4)		Proposed	
hydraulic diameter or radius	$r_h \equiv \frac{A_c L}{A_T}$	(1a)	$D_n \equiv \frac{4 A_F L}{A_b} = \frac{4 V}{A_b}$	(1b)
mass velocity	$G_c \equiv \frac{\omega}{A_c}$	(2a)	$G_n \equiv \frac{\omega}{A_F}$	(2b)
Reynolds number	$Re \equiv \frac{4 G_c r_h}{\mu}$	(3a)	$Re_n \equiv \frac{G_n D_n}{\mu}$	(3b)
friction factor	$f \equiv \frac{\Delta p_F}{\frac{L}{r_h} \frac{G_c^2}{2 \rho g_o}}$	(4a)	$f_n \equiv \frac{\Delta p_f}{4 \frac{L}{D_n} \frac{G_n^2}{2 \rho g_o}}$	(4b)
heat transfer coefficient	$h \equiv \frac{q/\eta_o A_T}{\Delta T}$ *	(5a)	$h_n \equiv \frac{q/A_b}{\Delta T}$	(5b)
Nusselt number	$Nu \equiv \frac{4 r_h h}{k}$	(6a)	$Nu_n \equiv \frac{h_n D_n}{k}$	(6b)

TABLE I. DEFINITIONS. (continued)

Quantity	Kays & London (4)	Proposed
Colburn j	$j \equiv \frac{h}{G_c\, c_p} (Pr)^{2/3}$ (7a)	$j_n \equiv \frac{h_n}{G_n\, c_p} (Pr)^{2/3}$ (7b)
	$* \eta_o \equiv 1 - \frac{A_f}{A_T} (1 - \eta_F)$ (8)	
	$\eta_f \equiv \frac{\tanh m\ell}{m\ell}$ (9)	
	$m \equiv \sqrt{\frac{2}{\delta} \frac{h}{k_m}}$ (10a) thin sheet fins	
	$m \equiv \sqrt{\frac{4}{\psi} \frac{h}{k_m}}$ (10b) circular pin fins	

a) Smooth Surface Passage



$A_F = ab$

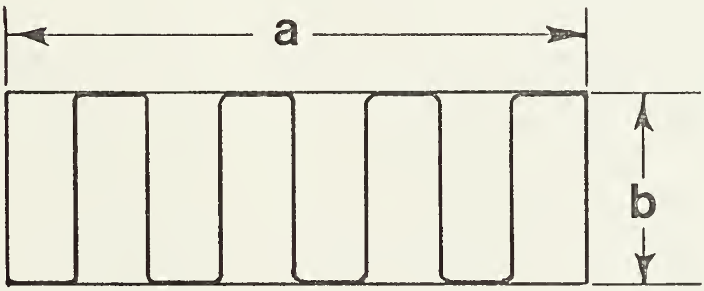
$A_b = 2aL$

$V = abL$

$$D_n = 4\frac{V}{A_b} = 4\frac{abL}{2aL} = 2b$$

$$G_n = \frac{\omega}{A_F} = \frac{\omega}{ab}$$

b) Plain Plate Fin Surface 6.2



$A_F = ab$

$A_b = 2aL$

$V = abL$

$$D_n = 4\frac{V}{A_b} = 4\frac{abL}{2aL} = ab$$

$$G_n = \frac{\omega}{A_F} = \frac{\omega}{ab}$$

Figure 1. Sample Calculation of Nonimal Diameter and Mass Flux for Rectangular Flow Passages

$$\frac{j_n}{j} = \frac{h_n}{h} \frac{G_c}{G_n} = \frac{\eta_o b}{2 r_h} \tag{16}$$

These ratios were used to convert K-L data to the proposed f_n and j_n vs. Re_n. Two assumptions were required in order to solve for the proper fin efficiency, η_f, required in the conversion. The heat exchanger used was constructed from aluminum ($k \simeq 100$ Btu/ft-hr-^oF) and the gas used was air at 90^oF. Figure 2 shows the two sets of curves on both basis for one of the K-L surfaces, namely their 6.2 plain plate-fin surface, Figure 61 of reference 4.

For any heat exchanger, the power per unit volume on one side is given by:

$$\frac{P}{V} = \frac{\omega \, \Delta p_f}{\rho \, V} \tag{17}$$

or from Table I,

$$\frac{P}{V} = \frac{2 \mu^3}{g_o \rho^2} \frac{f_n Re_n^3}{D_n^4} \tag{18}$$

For the same fluid at the same temperature level, μ and ρ are constant. Therefore:

$$\frac{P}{V} \propto f_n Re_n^3 / D_n^4 \tag{19}$$

The heat transfer for any heat exchanger is given by:

$$q = \varepsilon \, (T_{h,\,in} - T_{c,\,in}) \ \omega c_p \tag{20}$$

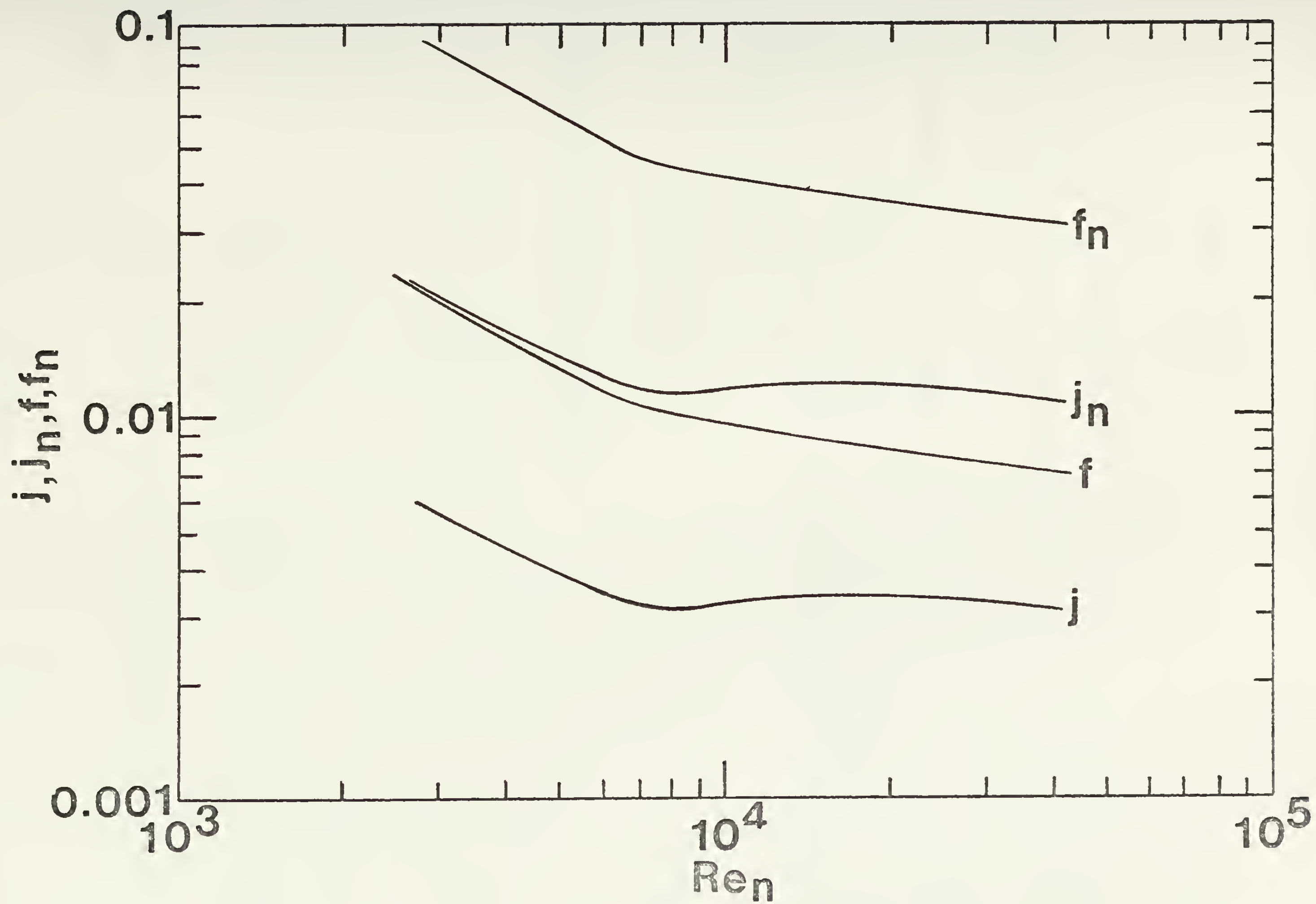


Figure 2. Comparison of Colburn j Factors (j and j_n), and Comparison of Friction Factors (f and f_n).

For any given flow arrangement, there exists a curve similar to that in Figure 3 which relates ε to NTU, where;

$$\mathrm{NTU} = \frac{A\ h_n}{\omega c_p} \tag{21}$$

This relationship between ε and NTU is always monotonically increasing. If fluid properties and flow rate are held constant, an increase in $A\ h_n$ results in an increase in NTU which results in an increase in ε and a higher heat transfer rate, q. Therefore, knowledge of either $A\ h_n$ or NTU will allow determination of the heat transfer rate. Thus the ratios $A\ h_n/V$ and NTU/V are quantities of interest from (21);

$$\frac{\mathrm{NTU}}{V} = \frac{A\ h_n}{V\ \omega c_p} \tag{22}$$

Substituting from Table I yields:

$$\frac{\mathrm{NTU}}{V} = \frac{4\ \mu}{Pr^{2/3}} \quad \frac{j_n\ Re_n}{\omega\ D_n^{\ 2}} \tag{23}$$

or,

$$\frac{A\ h_n}{V} = \frac{4\ c_p\ \mu}{Pr^{2/3}} \quad \frac{j_n\ Re_n}{D_n^{\ 2}} \tag{24}$$

For the same fluid at the same temperature levels, c_p, μ, and Pr are constant, therefore:

$$\frac{A\ h_n}{V} \propto \frac{j_n\ Re_n}{D_n^{\ 2}} \tag{25}$$

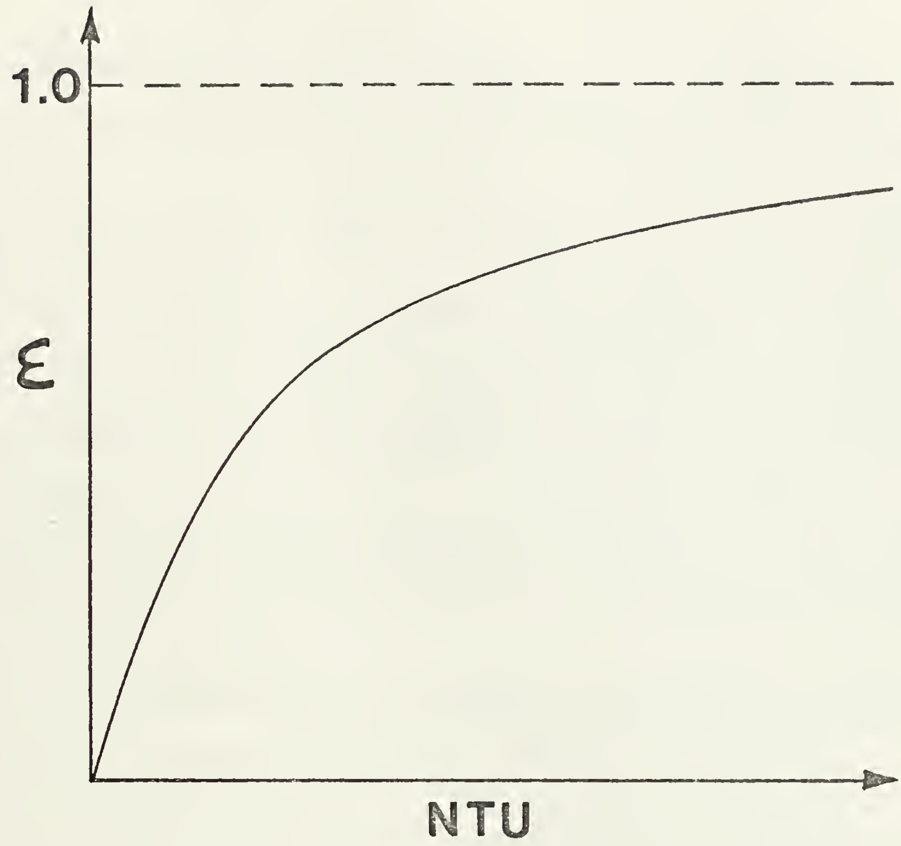


Figure 3. Typical Plot of Heat Exchanger Effectiveness, ε, vs. Number of Transfer Units, NTU.

and, here the flow rate, ω , is also constant:

$$\frac{NTU}{V} \propto \frac{j_n \, Re_n}{D_n^{\,2}} \tag{26}$$

With enhanced surface data in the form f_n vs. Re_n and j_n vs. Re_n , it is a simple matter to construct the performance parameters $f_n \, Re_n^{\,3}/D_n^{\,4}$ and $j_n \, Re_n/D_n^{\,2}$, and to plot them as in Figure 4. Different curves will result for different surfaces. Two surfaces shown by Figure 4 will be used to demonstrate the use of these curves in determining heat exchanger relative performance. All comparisons will be made for the same ω , $T_{c,\,in}$, $T_{h,\,in}$. This implies that any comparison which results in the same value for NTU/V will also have the same q/V.

Point o has been selected on surface 1 of Figure 4. It may lie anywhere on the surface 1 performance curve and represents a reference heat exchanger having the following specifications: P_o , NTU_o , q_o , L_o , $A_{F,\,o}$, V_o , where the subscript zero refers to surface 1. Diagramatically, the shape of the reference exchanger is labelled o in Figure 5.

Four different performance comparisons are immediately available from Figure 4 and are indicated by points a , b , c and d on surface 2. Here the curve for surface 2 lies above the curve for surface 1 in Figure 4.

Case a:

Same heat exchanger shape and volume ($L_a = L_o$, $V_a = V_o$, $A_{F,a} = A_{F,o}$).

This case represents a comparison of points o and a of Figure 4 where:

$$Re_{na} = Re_{no} \times \frac{D_{na}}{D_{no}}$$

Since the flow rate (ω) and the frontal area (A_F) are fixed. The results of this comparison are easily obtained as the ratios of the ordinate values and abscissa values and are shown in Figure 6a. Figure 5 shows the same heat exchanger shape for this case. Figure 6a shows the magnitude of the increase in pumping power and the heat transfer when using surface 2 instead of surface 1 in the same shape heat exchanger.

Case b:

Same heat exchanger volume and pumping power. ($V_b = V_o$, $P_b = P_o$).

This case represents a comparison of points o and b in Figure 4 since a vertical line in the performance plot has a fixed value of power per unit volume. This comparison yields the NTU ratio of the two heat exchangers and is easily obtained as the ratio of the ordinate values at o and b. Results are shown in Figure 6b and

shows the magnitude of the increase in the NTU for the same pumping power for surface 2 compared with surface 1. The relative shape for this case is shown in Figure 5. Since surface 2 has higher friction, the same pumping power will be obtained for a smaller Reynolds number, i.e. $Re_b < Re_o$. This causes the frontal area to increase, $A_{F,b} > A_{F,o}$, while the same volume requires the length to decrease, $L_b < L_o$.

Case c:

Same pumping power and number of transfer units. ($P_c = P_o$, $NTU_c = NTU_o$ (or $q_c = q_o$)).

This case represents a comparison of the heat exchanger size required for the same "job" - - a job being defined as the same pumping power resulting in the same heat transfer rate. This comparison is a line having a slope of 1 in Figure 4 (i.e. points o and c), since each axis is inversely proportional to the volume (NTU and P are constant). This results in the ratio of the heat exchanger volume required for the two surfaces obtained as the ratio of either the abscissas or the ordinates at points o and c. As long as surface 2 lies above surface 1, the result will be a smaller volume required to do the same "job". The resulting reduction in volume for surface 2 is shown in Figure 6c and the relative shape is shown in Figure 5.

Case d:

Same volume and number of transfer units. ($V_d = V_o$, $NTU_d = NTU_o$ (or $q_d = q_o$)).

This case compares the pumping power required by surface 2 to that required by surface 1 for the case when both heat exchangers yield the same overall heat transfer performance. It is obtained as the ratio of the abscissas of points o and d in Figure 4, since a horizontal line on the performance plot has a constant value of NTU/V. The results are illustrated in Figure 6d; they show the decreased pumping power required for surface 2. The surface 2 Reynolds number is the smallest of all four cases and consequently, the flow area is the largest. Since the volume is the same as the surface 1 exchanger volume, the length must decrease and the general shape is shown as d in Figure 5.

Thus for a very simply constructed plot of $j_n \; Re_n/D_n^2$ vs. $f_n \; Re_n^3/D_n^4$, it is possible to obtain useful performance comparisons between two heat exchangers for four different criteria.

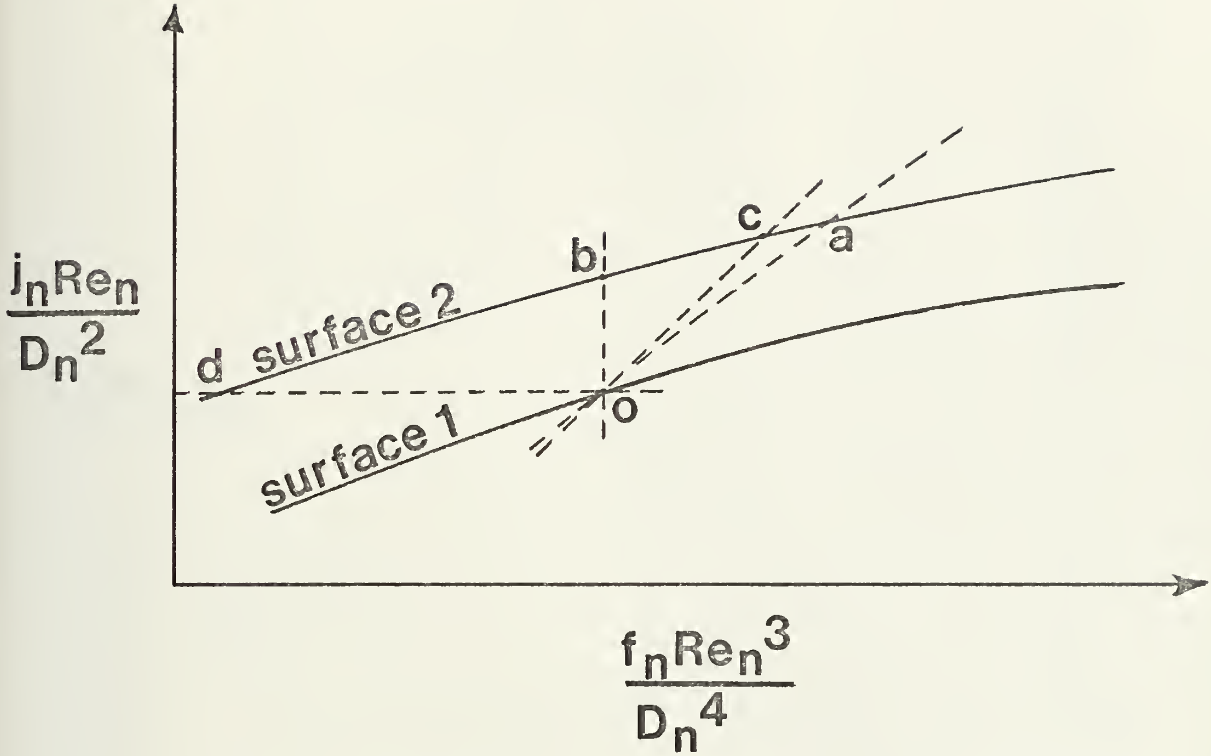


Figure 4. Performance Parameter Curves for Two Surfaces Showing Points Used in Sample Comparisons.

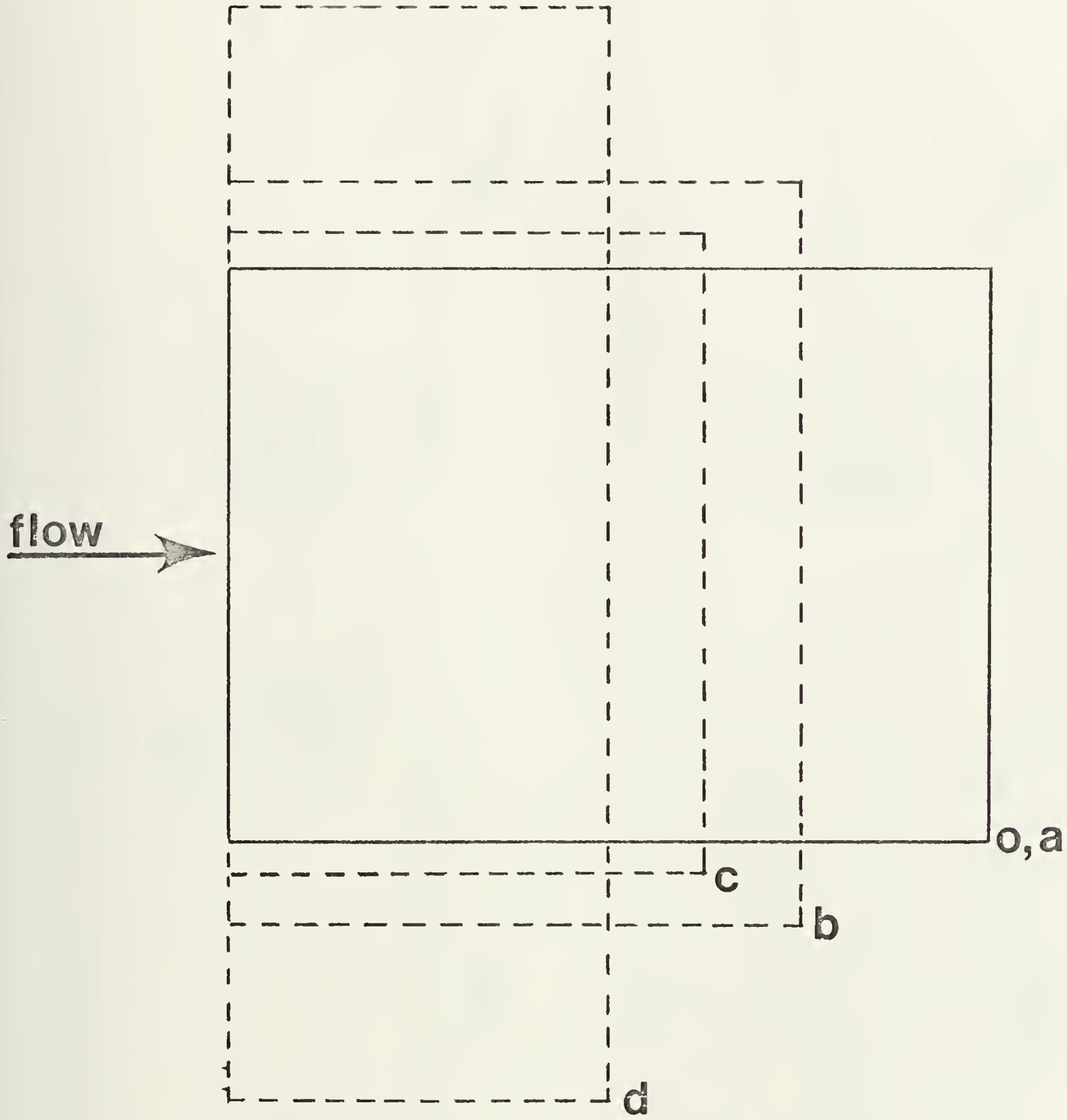


Figure 5. Relative Heat Exchanger Shape for Sample Comparisons Assuming Unit Height.

a) Case a: $L_a = L_o$, $V_a = V_o$, $A_{F,a} = A_{F,o}$

b) Case b: $P_b = P_o$, $V_b = V_o$

c) Case c: $P_c = P_o$, $NTU_c = NTU_o$, (or $q_c = q_o$)

d) Case d: $NTU_d = NTU_o$, (or $q_d = q_o$), $V_d = V_o$

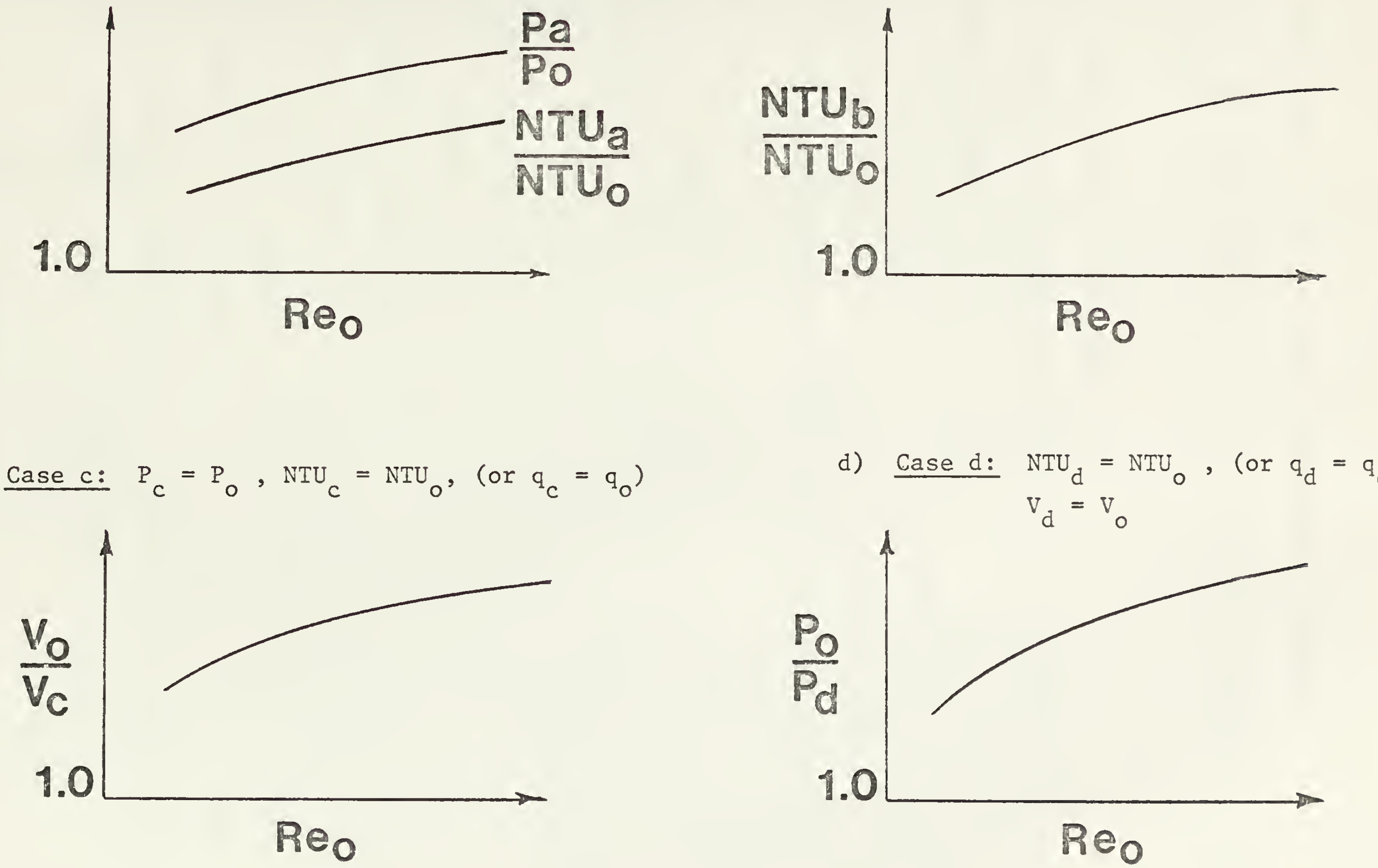


Figure 6. Typical Performance Comparison Results.

III. COMPARISON OF KAYS AND LONDON PLATE-FINNED SURFACES

All of the plate-fin surfaces were plotted as performance curves of $j_n Re_n / D_n^2$ vs. $f_n Re_n^3 / D_n^4$, for the same ω , $T_{c,\ in}$ and $T_{h,\ in}$. This is a plot equivalent to NTU/V vs. P/V.

K-L data are available for various finned surfaces having about nine different nominal diameters between 0.50 inches and 1.646 inches (b = 0.25 inches to 0.823 inches). For each nominal diameter, all of the finned surfaces were plotted as shown in Figure 7. However, in Figure 7, only the highest performance curve at each plate spacing is shown.

Table II lists all of the K-L plate-fin surfaces by type and surface designation. Due to the large number of surfaces used in the calculations, all Figures 7 through 17 will use the surface numbering system which is listed in the right hand column of Table II.

All of the plate fin surfaces were then compared to smooth surfaces having no enhancements and having the same nominal diameter (D_n) , (or plate spacing (b)), as the plate-fin surface being compared. This was accomplished using all plate-fin surfaces having a common nominal diameter of 0.50 inches (b = .25 inches). This was chosen due to the quantity of data available. There are 17 surfaces having this nominal diameter, whereas, the next nominal diameter (D_n = 0.82 inches) has data for only 3 different surfaces. The subscript e will be used to denote the enhanced surface and the subscript s will be used to denote the smooth surface.

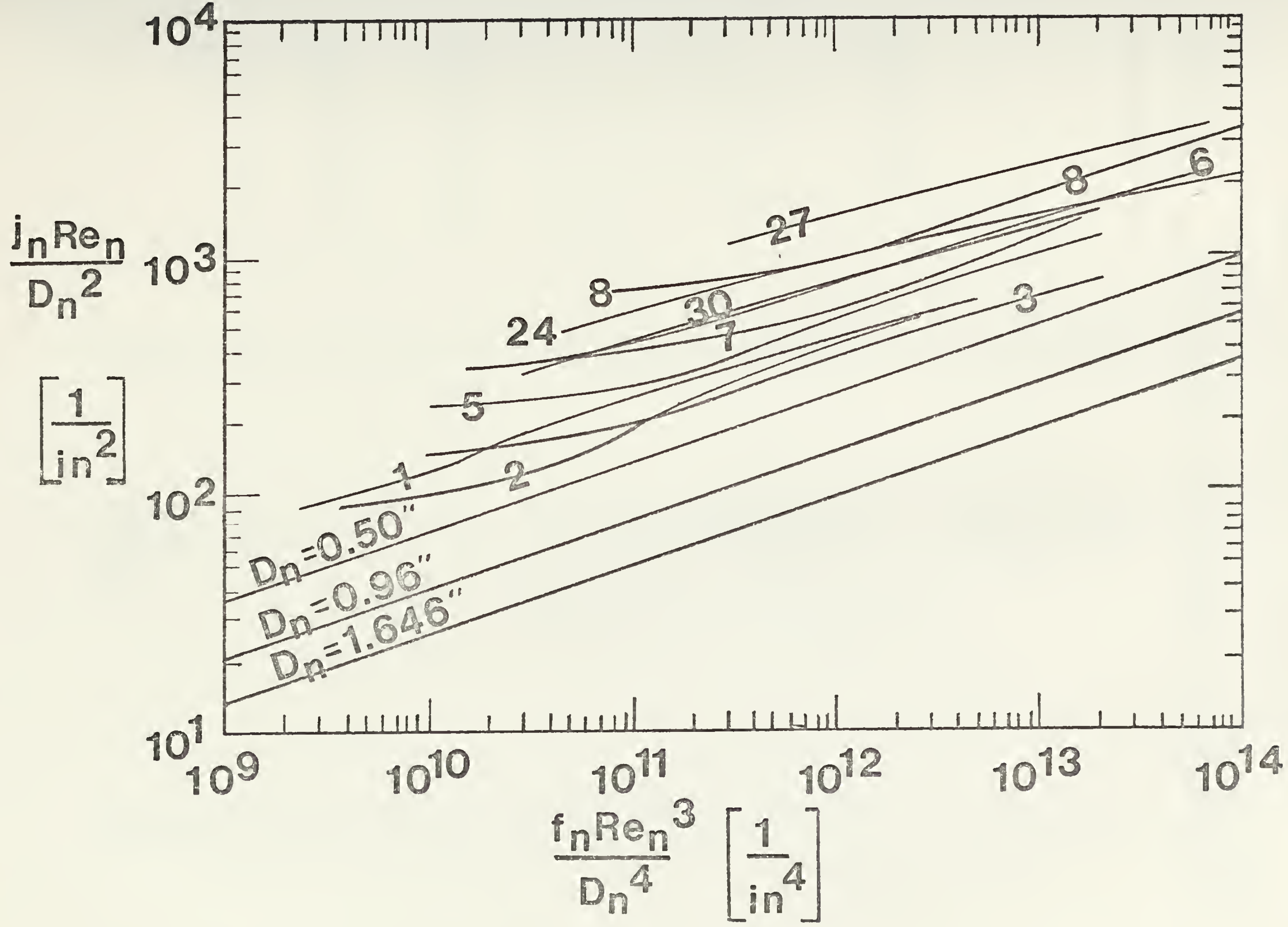


Figure 7. Performance Parameter Curve for Plate-Fin Surfaces.

NOTE: Only the best surface is shown for each plate spacing, and the pin-fin surfaces.

TABLE II. KAYS AND LONDON PLATE FIN SURFACES.

GENERAL SURFACE TYPE	PLATE SPACING (b) (inches)	SURFACE DESIGNATION	SURFACE NUMBERED IN FIGURES AS:
Plain plate-fin	.47	5.3	1
	.41	6.2	2
	.82	9.03	3
	.25	11.1	4
	.48	11.1(a)	5
	.33	14.77	6
	.42	15.08	7
	.25	19.86	8
Louvered plate-fin	.25	3/8 - 6.06	9
	.25	3/8(a) - 6.06	10
	.25	1/2 - 6.06	11
	.25	1/2(a) - 6.06	12
	.25	3/8 - 8.7	13
	.25	3/8(a) - 8.7	14
	.25	3/16 - 11.1	15
	.25	1/4 - 11.1	16
	.25	1/4(b) - 11.1	17
	.25	3/8 - 11.1	18
	.25	3/8(b) - 11.1	19
	.25	1/2 - 11.1	20
	.25	3/4 - 11.1	21
	.25	3/4(b) - 11.1	22
Strip-fin plate-fin	.25	1/4(s) - 11.1	23
	.49	3/32 - 12.22	24
	.41	1/8 - 15.2	25
wavy-fin plate-fin	.41	11.44 - 3/8W	26
	.41	17.8 - 3/8W	27
pin-fin plate-fin	.24	AP-1	28
	.40	AP-2	29
	.75	PF-3	30
	.50	PF-4(F)	31
	.51	PF-9(F)	32

Smooth surface $j_n \, Re_{ns}/D_{ns}^{2}$ calculations were completed utilizing the Colburn correlation for forced-convection, turbulent flow in tubes with the appropriate nominal diameter (D_n) from reference 8. This reduces to:

$$j_s \equiv 0.023 \, Re^{-0.2} \tag{27}$$

For a smooth surface $h = h_n$, $G = G_n$, $Re = Re_n$.

Then:

$$j_s \, Re_n = 0.023 \, Re_n^{0.8} \tag{28}$$

Smooth surface $f_n \, Re_n^{3}/D_n^{4}$ was calculated using the linear approximation:

$$f_s \cong .0791 \, Re_n^{-0.25} \tag{29}$$

For a smooth surface $f_n = f$.

Thus:

$$f_s \, Re_n^{3} = .0791 \, Re_n^{2.75} \tag{30}$$

Combining (28 and 30) to obtain the equation for the smooth surface leads to:

$$j_s \, Re_n = .0481 \, (f_s \, Re_n^{3})^{0.291} \tag{31}$$

or:

$$\frac{j_s \, Re_n}{D_n^{2}} = \frac{.0481}{D_n^{.836}} \left[\frac{f_s \, Re_n^{3}}{D_n^{4}} \right]^{0.291} \tag{32}$$

These lines were calculated for all different nominal diameters. However, only three separate nominal diameters (0.50" , 0.94" , 1.646") which span the range of values are shown on Figure 7 as the three straight lines at the bottom of the figure.

Cases a through d were then calculated for all plate-fin surfaces having a nominal diameter of 0.50 inches and were compared with a smooth surface having a nominal diameter (D_n) equal to 0.50 inches. Figures 8 and 9 show the case for same Re_n since the nominal diameter is fixed. Figure 8 shows the ratio of ordinates (NTU_e/NTU_s) vs. Re_n and Figure 9 shows the ratio of abscissas (P_e/P_s) vs. Re_n. Note that a larger ratio on Figure 8 is preferred and a smaller ratio on Figure 9 is preferred.

Figure 10 shows the case for P = constant, V = constant or case b (i.e. NTU_e/NTU_s vs. Re_n).

Figure 11 shows the case NTU = constant, P = constant or case c (i.e. V_s/V_e vs. Re_n).

Figure 12 shows the case NTU = constant, V = constant or case d (i.e. P_s/P_e vs. Re_n).

For Figures 10 through 12 higher ratios are preferred.

With the completion of the performance comparison for a nominal diameter of 0.50 inches all of the plate fin surfaces of K-L (4) of all diameters will now be compared to a smooth surface having a nominal

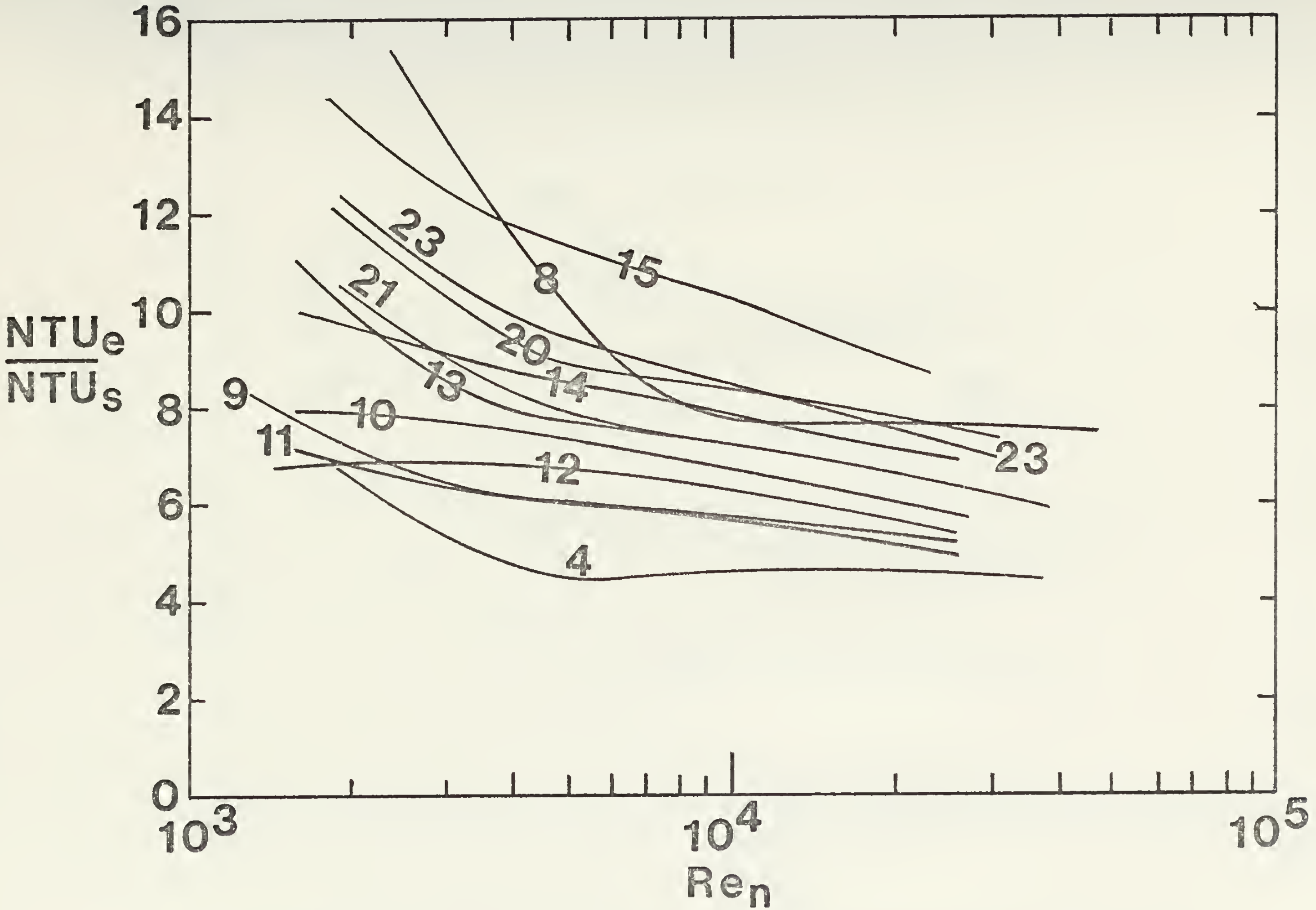


Figure 8. Performance Comparison Results (NTU_e/NTU_s) for Case a (i.e. same shape and V). D_n = 0.50" for both the enhanced and plain surface. $\therefore$ Re_n = constant.

NOTE: Surfaces numbered from TABLE II as 16-19 and 22 all lie between surfaces numbered 15 and 20. Surface number 22 follows surface number 21.

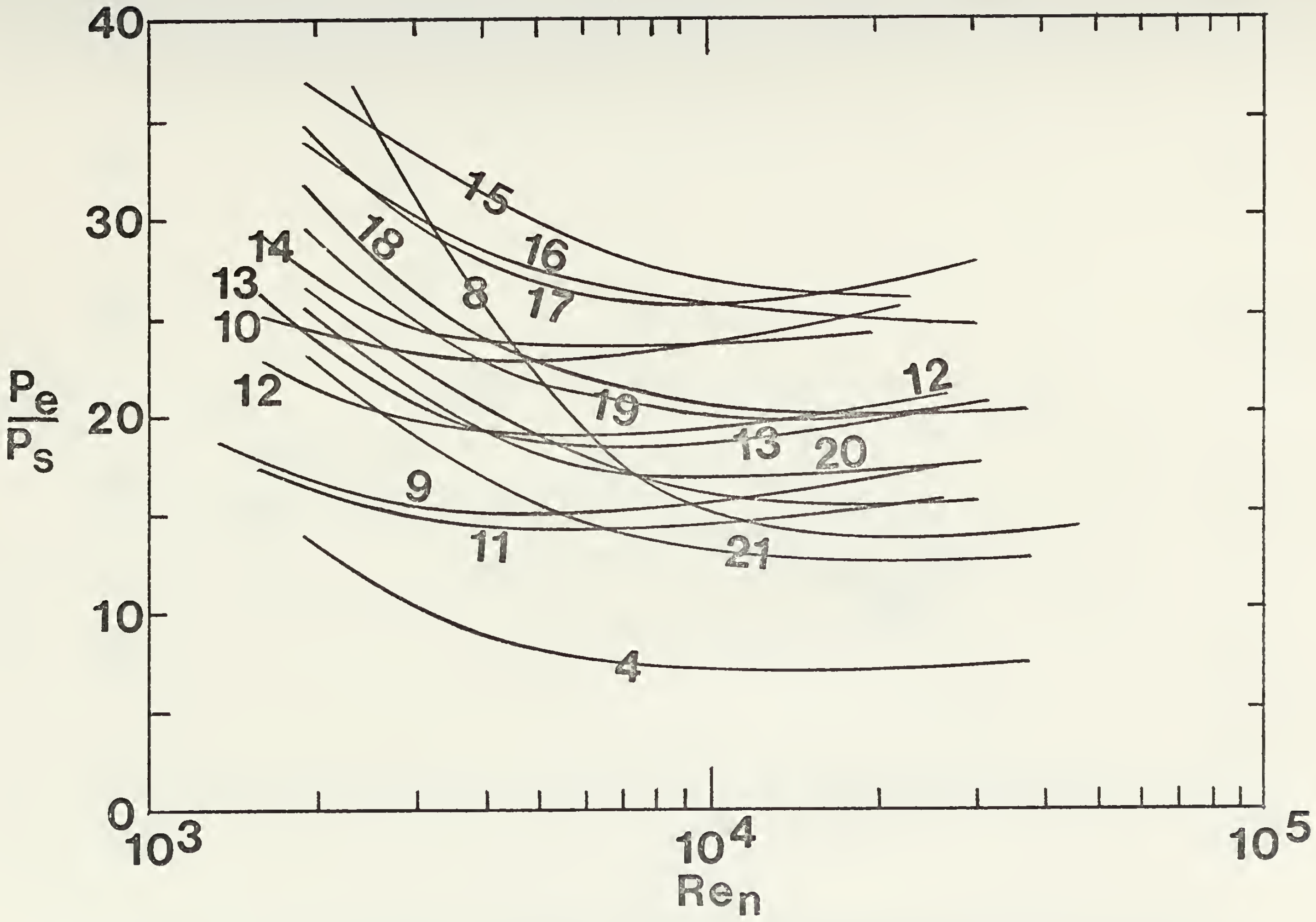


Figure 9. Performance Comparison Results (P_e/P_s) for Case a (i.e. same shape and V = constant). D_n = 0.50" for both the enhanced and plain surface. ∴ Re_n constant.

NOTE: Surface Number 22 from TABLE II follows surface number 21.

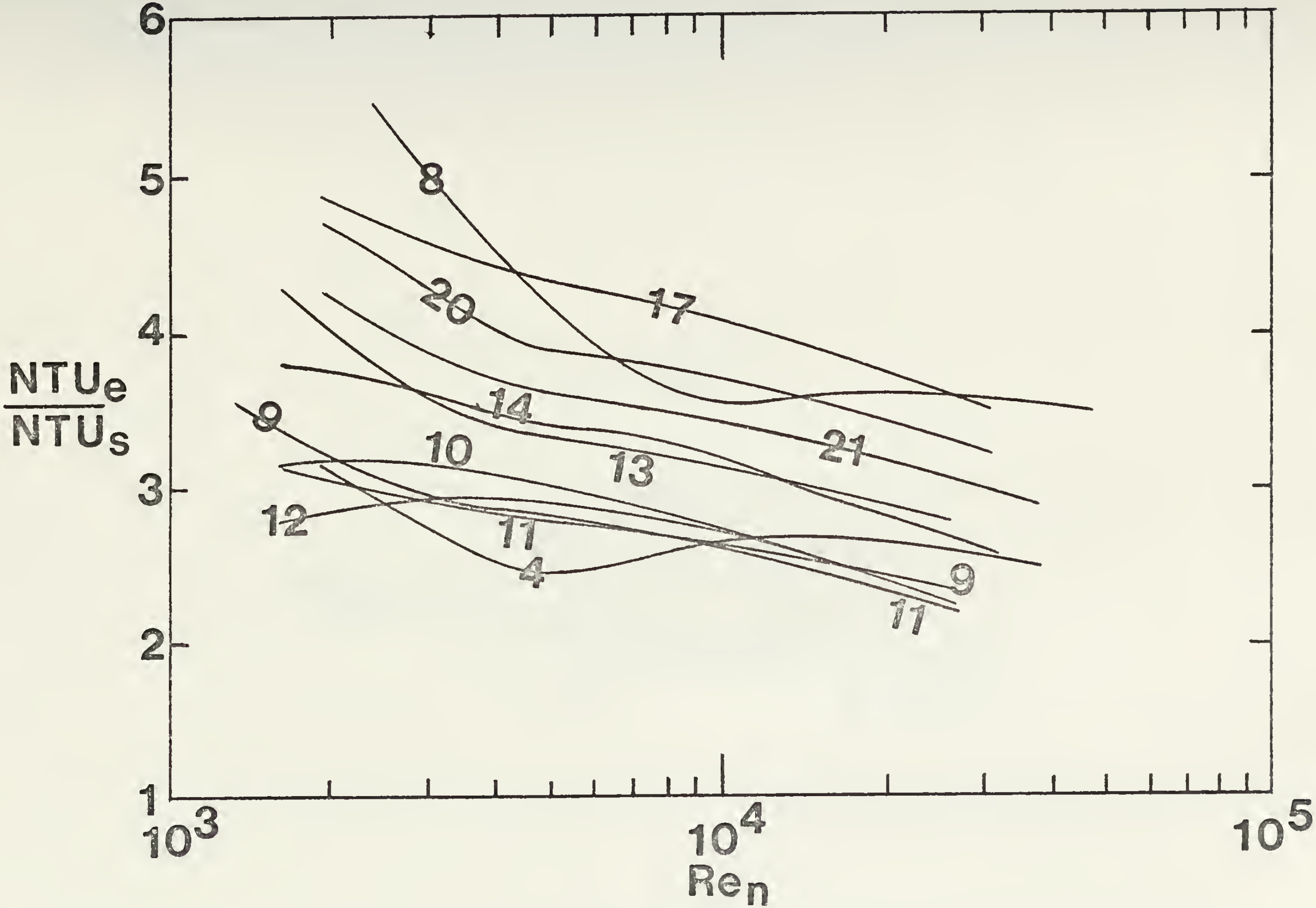


Figure 10. Performance Comparison Results (NTU_e/NTU_s) for Case b (i.e. V = constant, P = constant). D_n = 0.50" for both enhanced and plain surfaces.

NOTE: Surfaces numbered from TABLE II as 15, 16, 18, 19, 22, and 23 all lie between surfaces numbered 17 and 20.

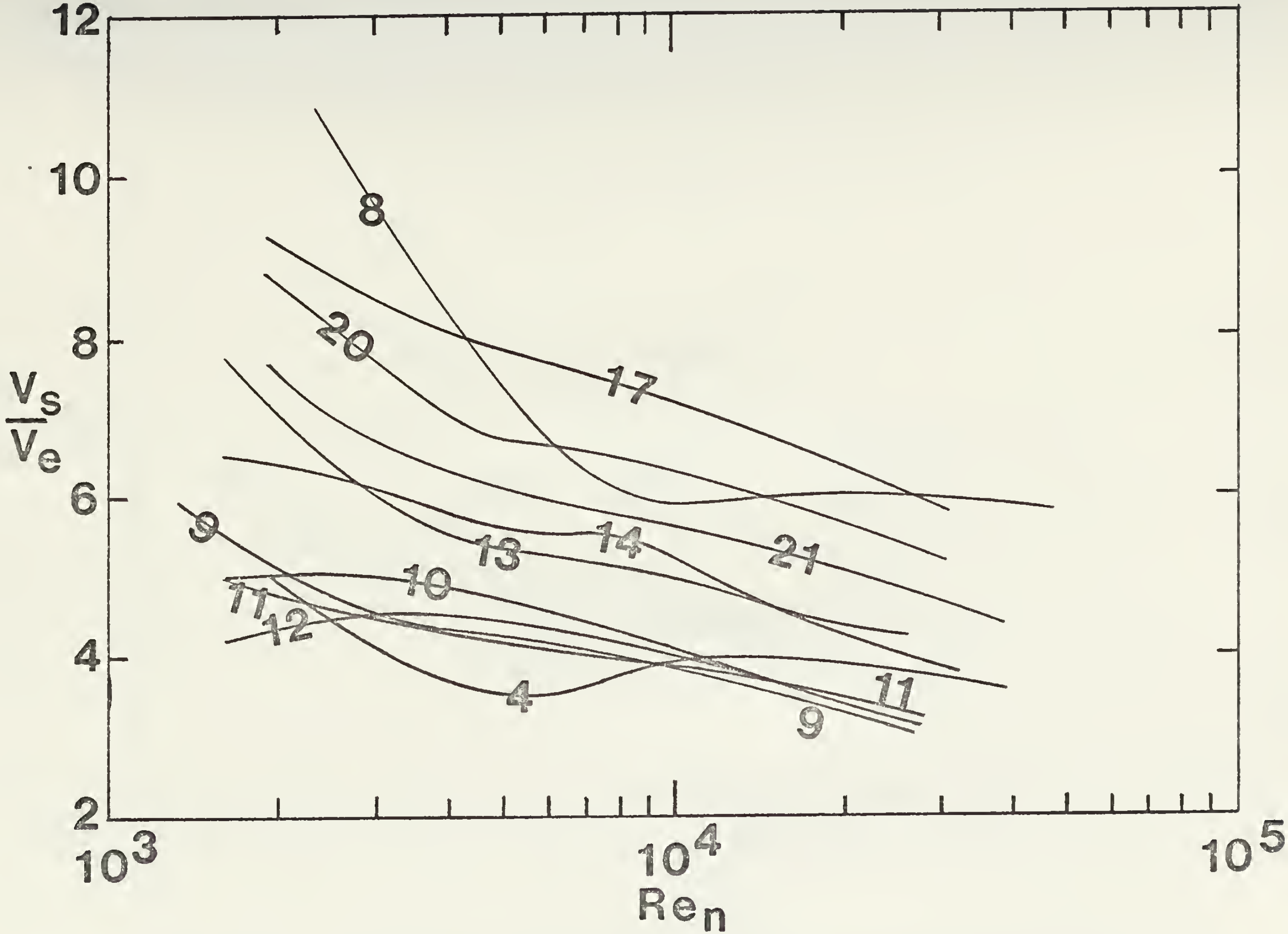


Figure 11. Performance Comparison Results (V_s/V_e) for Case c (i.e. NTU = constant, P = constant). D_n = 0.50" for both enhanced and plain surface.

NOTE: Surfaces numbered from TABLE II as 15, 16, 18, 19, 22, 23 all lie between surfaces numbered 17 and 20.

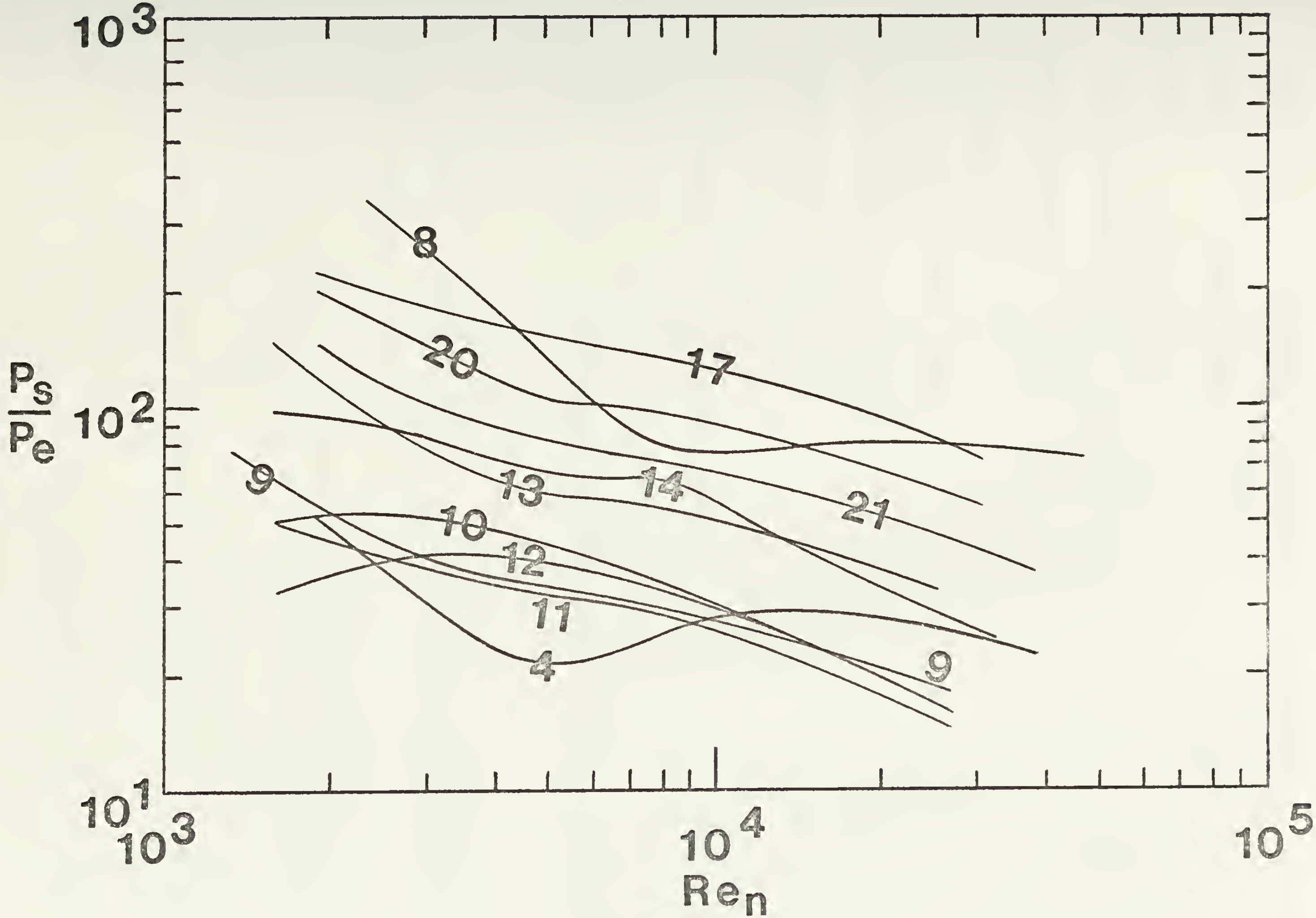


Figure 12. Performance Comparison Results (P_s/P_e) for Case d (i.e. NTU = constant, V = constant). D_n = 0.50" for both enhanced and plain surfaces.

NOTE: Surfaces numbered from TABLE II as 15, 16, 18, 19, 22, 23 all lie between surfaces numbered 17 and 20.

diameter of 0.50 inches. Comparison of each individual surface to a smooth surface having the same nominal diameter as the enhanced surface is the best type of comparison, if the comparison is undertaken for surfaces having a common nominal diameter. When the plate spacing varies, this comparison method is invalid. Comparison of many surfaces to a single common smooth plate nominal diameter permits a relative comparison between surfaces having different nominal diameters. Through use of this method, all plate-fin surfaces from K-L (4) will be considered. Since a nominal diameter of 0.50 inches has been fully investigated, only those D_n = .50 inch surfaces that bound the maximum or minima for each case will be shown in Figures 13 through 17.

Figures 13 and 14 show the case for the same shape and V = constant or case a. Figure 13 shows the ratio of ordinates (NTU_e/NTU_s) vs. Re_{ns} and Figure 14 shows the ratio of abscissas (P_e/P_s) vs. Re_{ns}.

Figure 15 shows the case P = constant, V = constant or case b (i.e. NTU_e/NTU_s vs. Re_n).

Figure 16 shows the case NTU = constant, P = constant or case c (i.e. V_s/V_e vs. Re_n).

Figure 17 shows the case NTU = constant, V = constant or case d (i.e. P_s/P_e vs. Re_n).

Higher ratios are preferred for Figures 13, 15, 16 and 17 while a lower ratio is preferred for Figure 14.

Calculations were also made for the five pin-fin surfaces of reference 4. Only the best of these surfaces is shown in Figure 7. They are not shown in any of Figures 8 through 17. Since the best four have the same performance despite having different nominal diameters. Surfaces 28, 29, 31 all follow surface 30 of Figure 7 while surface 32 lies atop surface 1.

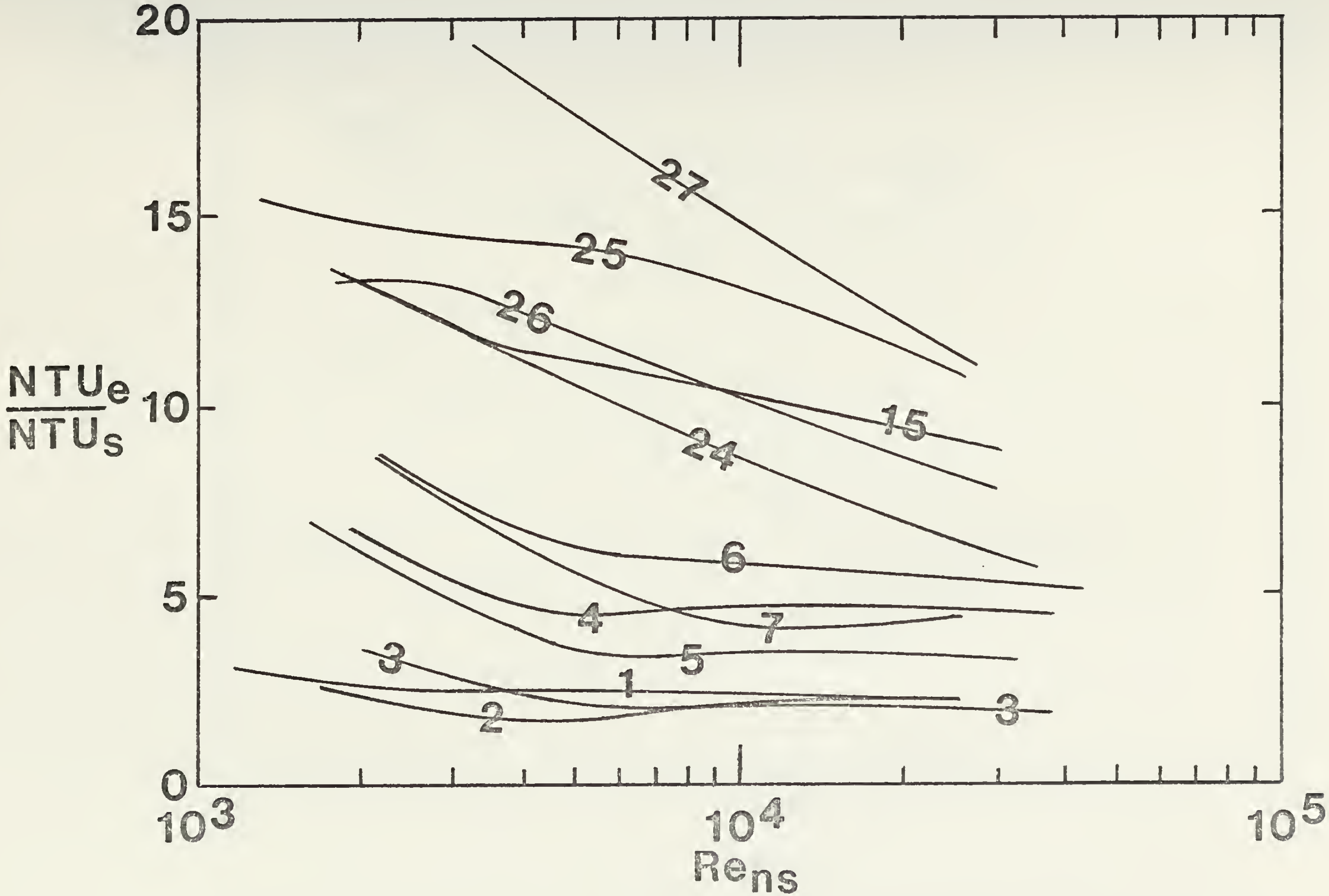


Figure 13. Performance Comparison Results (NTU_e/NTU_s) for Case a (i.e. same shape and volume) to Smooth Surface with D_n = 0.50".

NOTE: Only the maximal and minimal are shown for plate-fin surfaces with D_n = 0.50" (curve numbers 15, and 4 respectively).

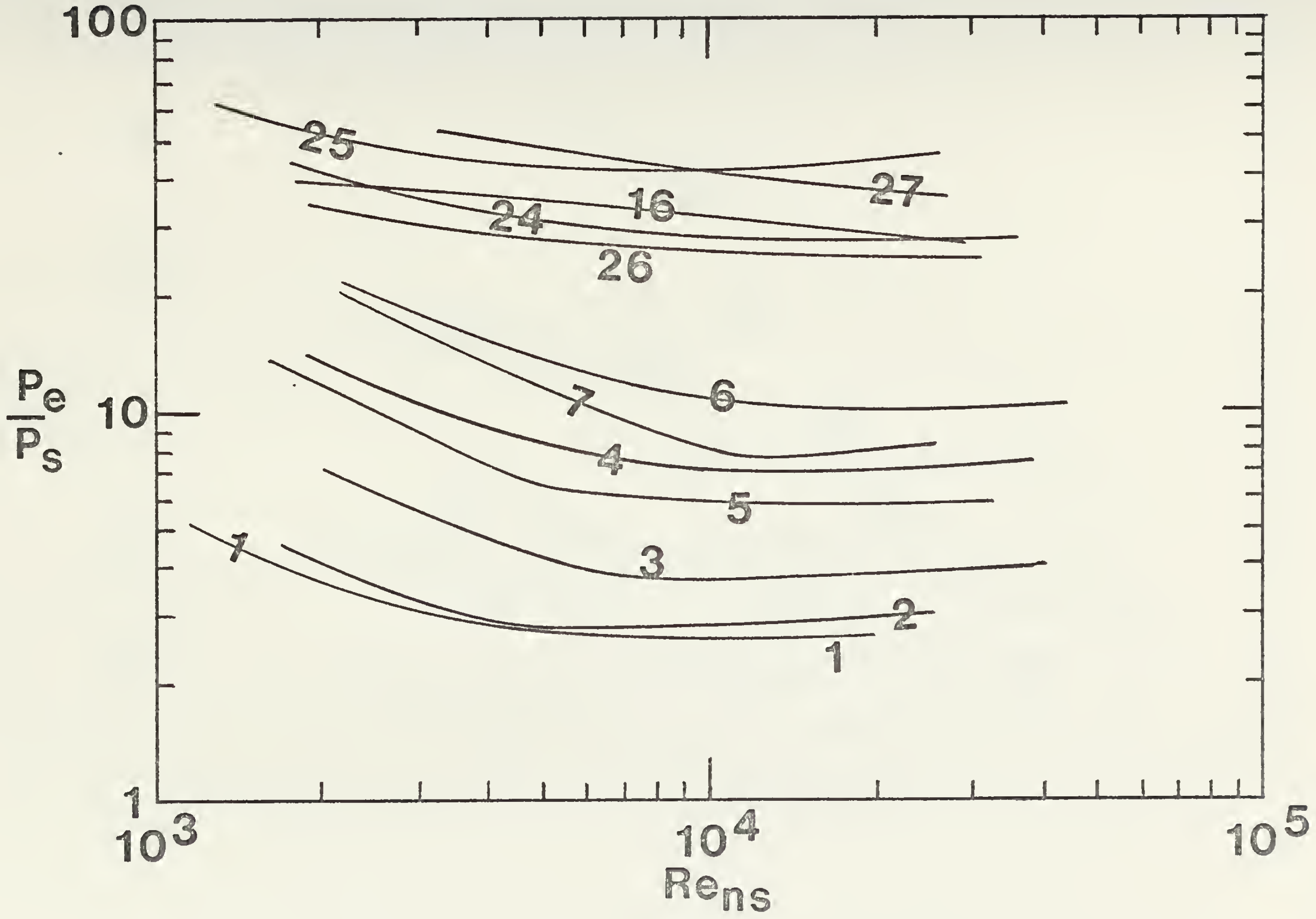


Figure 14. Performance Comparison Results (P_e/P_s) for Case a (i.e. same shape and volume) to Smooth Surface with D_n = 0.50".

NOTE: Only the maximal and minimal are shown for plate-fin surfaces with D_n = 0.50" (curve numbers 16 and 4 respectively).

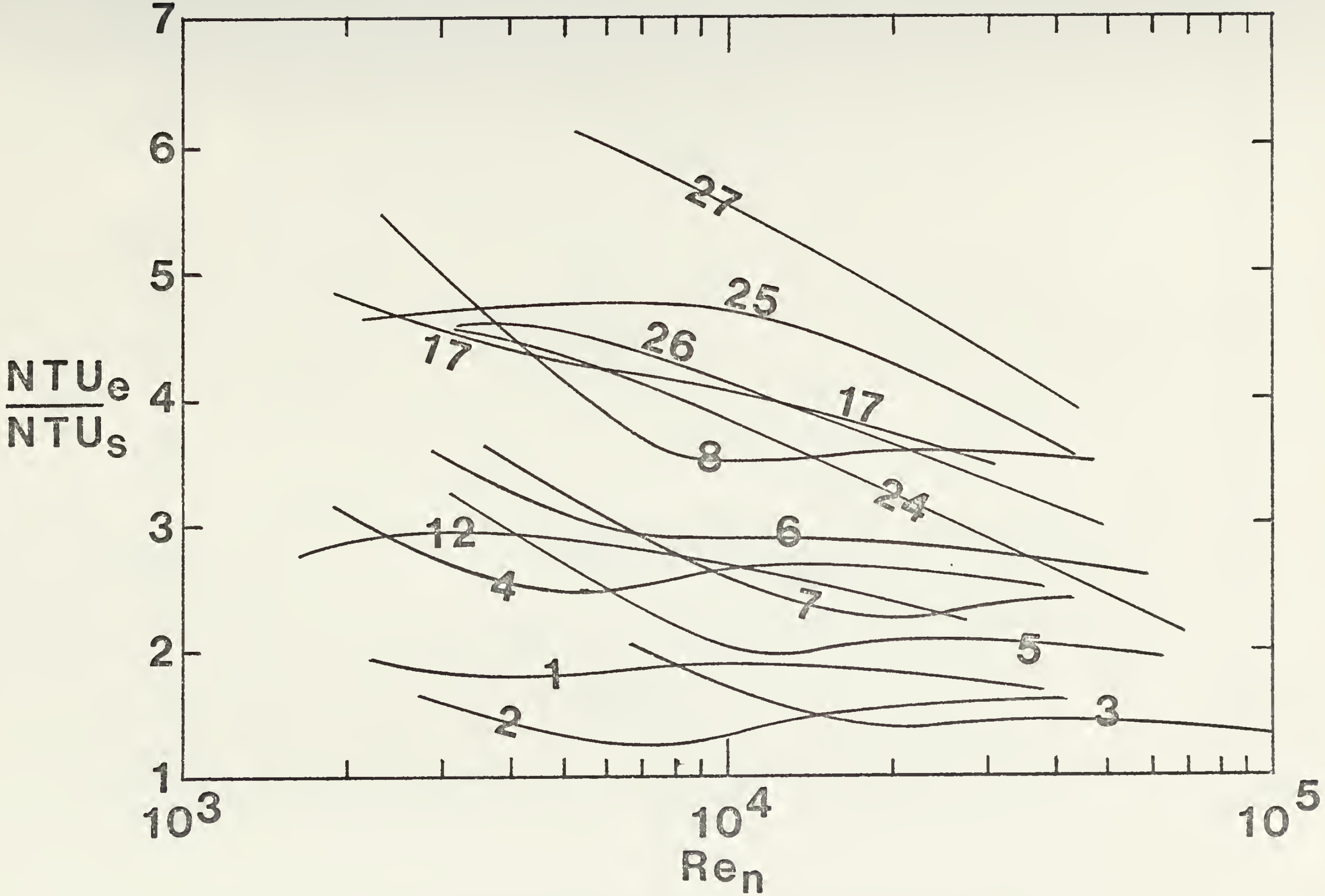


Figure 15. Performance Comparison Results (NTU_e/NTU_s) for Case b (i.e. P = constant, V = constant) to Smooth Surface with D_n = 0.50".

NOTE: Only the maxima and minima are shown for plate-fin surfaces with D_n = 0.50" (curve numbers 8, 17 and 4, 12 respectively).

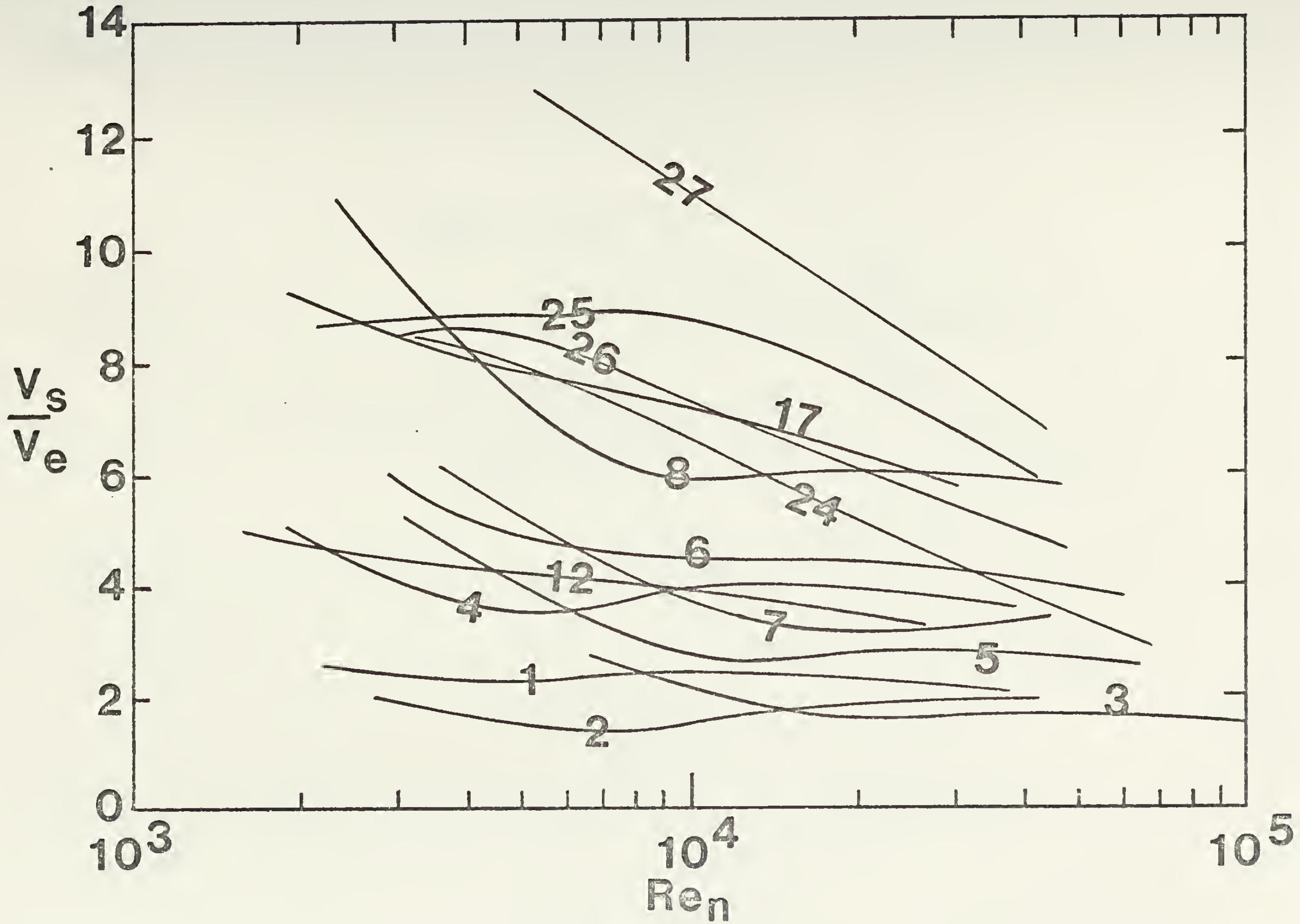


Figure 16. Performance Comparison Results (V_s/V_e) for Case c (i.e. NTU = constant, P = constant) to Smooth Surface with D_n = 0.50".

NOTE: Only the maxima and minima are shown for plate-fin surfaces with D_n = 0.50" (curve numbers 8, 17 and 4, 12 respectively).

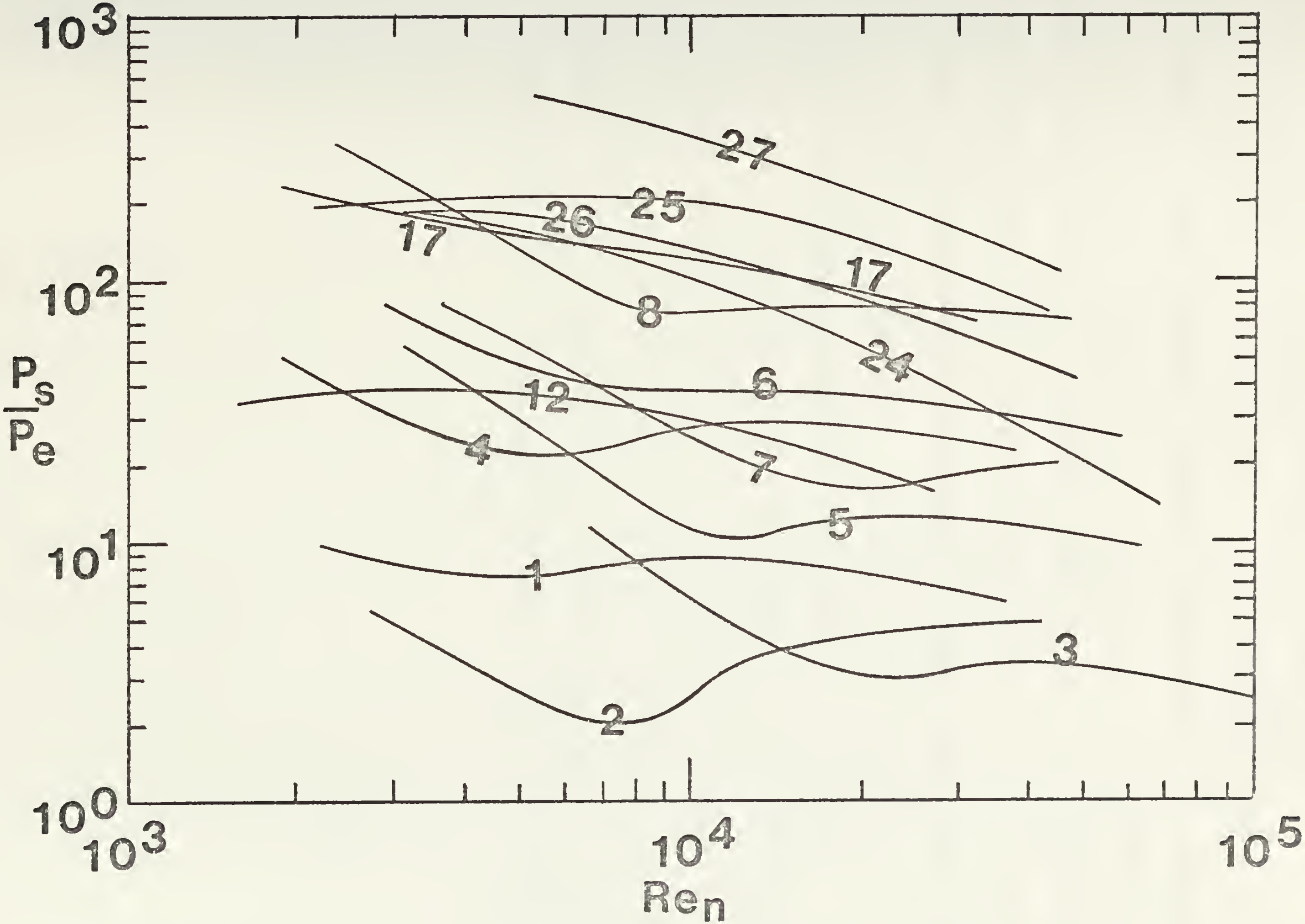


Figure 17. Performance Comparison Results (P_s/P_e) for Case d (i.e. NTU = constant, V = constant) to Smooth Surface with D_n = 0.50".

NOTE: Only the maxima and minima are shown for plate-fin surfaces with D_n = 0.50" (curve numbers 8, 17 and 4, 12 respectively).

IV. COMPARISON WITH SAND GRAINED ROUGHENED SURFACES

Many enhanced surfaces have been designed to increase the performance of heat exchangers. Dipprey and Saberski (3) determined the heat transfer coefficient and friction factor experimentally for rough tubes containing a granular type surface with roughness-height-to-diameter ratios (e/D) ranging from 0.0024 to 0.049. The best of these surfaces from the standpoint of higher heat transfer coefficients for lower friction factors (i.e. e/D = 0.049) was converted to the proposed performance parameters and plotted as $j_n\ Re_n$ vs. $f_n\ Re_n^3$ in reference 11. Figure 18 contains a plot of the performance parameters $j_n\ Re_n/D_n^2$ vs. $f_n\ Re_n^3/D_n^4$ for the Dipprey and Saberski surface using $D_n = 0.50$ as well as the best two K-L plate-finned surfaces having $D_n = 0.82$ inches and 0.50 inches respectively. Also shown is the smooth surface performance curve for $D_n = 0.50$ inches.

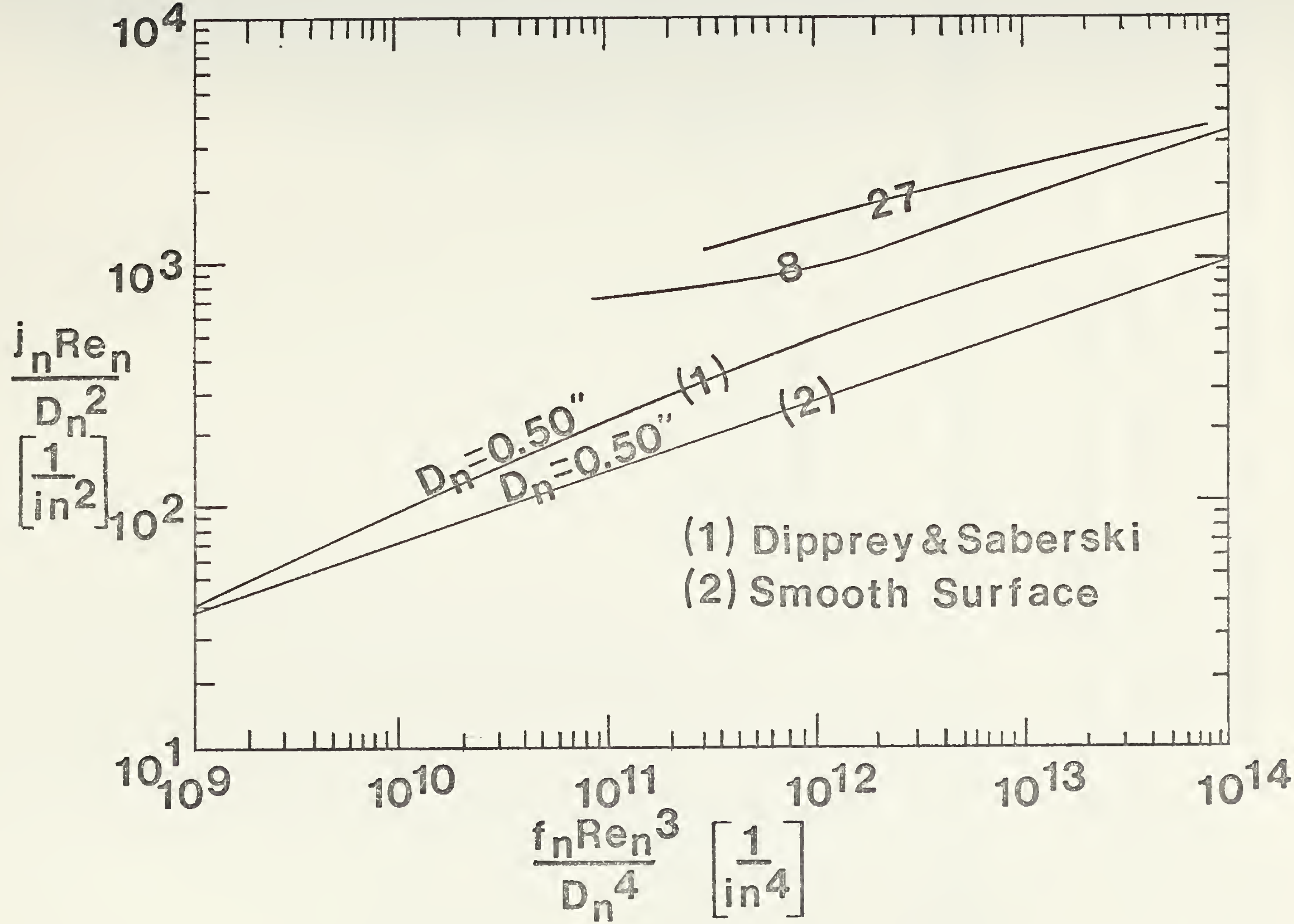


Figure 18, Performance Parameter Curves for Best Plate-fin Surfaces and Dipprey & Saberski [3] e/D = .049 surface.

V. RESULTS OF COMPARISON

The higher a curve lies on the performance curve of $j_n \ Re_n/D_n^{\ 2}$ vs. $f_n \ Re_n^{\ 3}/D_n^{\ 4}$ the better the surface performance. Inspection of the curves for all surfaces suggests that the following surfaces, or groups of surfaces are best in order of decreasing performance.

Ranking Number	Surface Number	Plate Spacing
1	27	.41
2	25	.41
3-11	8, 15-20, 23, 26	.25(8), .41(1)
12-14	21, 22, 24	.25(2), .49(1)
15-20	13, 14, 28-31	.24(1), .25(2), .40(1), .50(1) .75(1)
21	6	.33
22-27	4, 7, 9-12	.25(5), .42(1)
28	5	.48
29-30	1, 32	.47(1), .51(1)
31-32	2, 3	.41(1), .82(1)

VI. CONCLUSIONS

1. The comparison method of La Haye et al. (5) was modified to compare heat exchanger performance on four different bases:

   a. Same shape and volume of heat exchanger.

   b. Same exchanger volume and pumping power.

   c. Same pumping power and NTU

   d. Same volume and NTU

2. All of Kays-London plate finned surfaces, unfinned surfaces and a sand roughened surface were compared on the above bases.

3. The "best" surface of those compared is found to be the wavy-fin plate-fin 17.8 - 3/8 W with plate spacing of 0.41 inches.

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